

# Thermophoretic Effect on Double Diffusive Convective Flow of a Chemically Reacting Fluid over a Rotating Cone in Porous Medium

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**Abstract**—In this paper, we analyzed buoyancy driven convective heat and mass transfer flow of a chemically reacting fluid over a rotating cone in porous medium with thermophoretic particle deposition. Darcy law is used to describe the fluid flow in porous medium. The governing equations for mass, momentum, energy and concentration equations are transformed into non-dimensional nonlinear ordinary differential equations using specified similarity transformations and then solved by employing shooting method with 4<sup>th</sup> Runge-Kutta method. The numerical results are reported graphically for various physical parameters, namely chemical reaction parameter, thermophoretic constant and thermophoresis parameter on tangential, circumferential and normal velocity profiles as well as tangential and azimuthal skin-friction coefficient and rate of heat and mass transfer and thermophoretic particle deposition velocity near the cone surface.

**Index Terms:** Rotating Cone, Porous Medium, Thermophoresis, Chemical Reaction.

## 1 INTRODUCTION

The study of combined heat and mass transfer problems has great importance in extending theory of separation processes, chemical and hydro-metallurgical industries. Heat transfer phenomenon in addition to the mass transfer has received great attention of modern researchers for its enormous applications in chemical industries, reservoir engineering and other processes. A few representation fields of this type of flows in which combined heat and mass transfer with chemical reaction effect plays an important role, are electrical energy is extracted directly from a moving conducting fluid, formation and dispersion of fog, damage of crops due to freezing, distribution of temperature and moisture over groves of fruit trees, manufacturing of ceramics, polymer production.

Combined heat and mass transfer in rotating systems problems are of importance in many processes. These processes take place in many industrial, geothermal and engineering applications, such as nuclear plants, vortex chambers, filtration, shielding of a rotating bodies, design of rotating machinery, oceanic circulations, gas turbines, various propulsion devices for space vehicles, missiles, satellites and aircraft, the design of turbines and turbo machines, in estimating the flight path of rotating wheels and spin-stabilized missiles and in the modeling of many geophysical vortices. At first, Tien [1] analyzed free convective heat transfer from a rotating cone. Hering and Grosh [2] studied in mixed convection and reported the results by dividing the regimes of flow as purely free, forced and combined convection flows. Himasekhar and Sarma [3] analyzed convective heat transfer flow over a rotating cone with suction effect. Chamkha [4] used Darcy's

model to study convective heat transfer flow past a rotating cone with heat generation and magnetic effect. Recently, Anilkumar and Roy [5, 6] investigated unsteady mixed convection from a rotating cone in a rotating fluid. Unsteady MHD convective flow in the stagnation region of a rotating sphere at constant wall temperature and heat flux conditions is investigated by Chamkha et.al [7]. Chamkha and Rashad [8] studied Chemical reaction and cross diffusion effects on unsteady convective flow over a rotating cone.

Thermophoresis is the migration of aerosol and other particles in the direction of a reduction of the temperature gradient. Such a phenomenon encountered in the engineering applications in aerosol reactors, heat exchanger fouling, optical fiber production etc. In optical fiber synthesis, thermophoresis is identified as the mass transfer mechanism as used in the modified chemical vapor deposition (MCVD). Chamkha [9, 10] studied thermophoretic effect on hydromagnetic convective flow over a flat surface, vertical cylinder, in a fluid saturated porous medium. Partha [11, 12] analyzed thermophoresis particle deposition on natural convective in a non-Darcy porous medium with Soret and Dufour effects under permeable and impermeable conditions. Rahman and Postelnicu [13] investigated thermophoresis effect on forced convective flow of a viscous fluid due to rotating disk. Bakier and Gorla [14] studied thermal radiation effect on convective flow over a semi-infinite vertical plate with thermophoretic effect. Muhaimin et.al [15] investigated impact of thermophoretic particle deposition on unsteady convective flow over a porous wedge in non-Darcy porous medium with chemical reaction and temperature-dependent viscosity. Recently, Kameswaran et.al [16] studied non-linear convection and thermophoresis on convective boundary layer flow over a vertical wall in a non-Darcy porous medium. Bhuvanavijaya and Mallikarjuna [17] investigated the effects of variable properties and thermophoretic on convective heat and mass

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transfer flow over vertical wavy surface in a fluid saturated porous medium.

In view of the above applications, authors envisage to investigate thermophoresis particle deposition on buoyancy driven mixed convective heat and mass transfer flow over an impermeable vertical rotating cone in a fluid saturated porous medium with chemical reaction. The following strategy is pursued in the rest of the paper. The problem is formulated in section-2. Section-3 contains the numerical solution. Results and discussions are presented in section-4. The concluding remarks are given in section-5.

## 2 FORMULATION OF THE PROBLEM

We consider the steady boundary layer flow of axisymmetric, incompressible chemically reacting fluid past an infinite rotating vertical cone in a fluid saturated porous medium. Fig-1 shows physical configuration and coordinate of the system. Let the vertical cone is assumed to have variable temperature and variable concentration which are higher than free stream temperature and concentration. Consider the rectangular curvilinear coordinate system. Let  $u, v$  and  $w$  be tangential ( $x$ ), circumferential ( $y$ ) and normal velocities ( $z$ ) respectively. The fluid saturated porous medium is assumed to be homogeneous and isotropic which is in locally thermodynamic equilibrium with solid matrix. The physical properties of the fluid are assumed to be constant except density in the buoyancy force term. The buoyancy force is made up of two components; the fluid temperature and concentration variations. In addition thermal absorption is assumed in energy equation. Using the above assumptions and boundary layer approximations, the governing equations, continuity, Darcy equation (using linear Boussinesq approximation and after eliminating pressure), energy and concentration equation can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} + \frac{u}{x} = 0 \quad (1)$$

$$\rho \left( u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} - \frac{v^2}{x} \right) = \mu \frac{\partial^2 u}{\partial z^2} - \frac{\mu}{K} u + \rho g \beta_T (T - T_\infty) \cos(\alpha) + \rho g \beta_c (C - C_\infty) \cos(\alpha) \quad (2)$$

$$\rho \left( u \frac{\partial v}{\partial x} + w \frac{\partial v}{\partial z} + \frac{uv}{x} \right) = \mu \frac{\partial^2 v}{\partial z^2} - \frac{\mu}{K} v \quad (3)$$

$$\left( u \frac{\partial T}{\partial x} + w \frac{\partial T}{\partial z} \right) = \frac{k_e}{\rho c_p} \frac{\partial^2 T}{\partial z^2} + Q(C - C_\infty) \quad (4)$$

$$u \frac{\partial C}{\partial x} + w \frac{\partial C}{\partial z} + \frac{\partial}{\partial z} (C v_t) = D \frac{\partial^2 C}{\partial z^2} - k_r (C - C_\infty) \quad (5)$$

The corresponding boundary conditions are

$$\begin{aligned} u=0, v=r\Omega, w=0, T=T_w(x), C=C_w(x) \quad \text{at } z=0 \\ u=0, v=0, T=T_\infty, C=C_\infty \quad \text{as } z \rightarrow \infty \end{aligned} \quad (6)$$

where  $u, v$  and  $w$  are tangential, circumferential and normal velocity components along  $x$ -axis (meridional section),  $y$ -axis (circular section) and the  $z$ -axis (normal to the cone surface) directions respectively,  $\Omega$  is the angular velocity of the rotation,  $\rho$  is the fluid density,  $\mu$  is the

dynamic viscosity,  $g$  is the acceleration due to gravity,  $\beta_T$  and  $\beta_c$  are the thermal and concentration coefficients,  $\alpha$  is the cone apex half angle,  $K$  is the permeability of the porous medium,  $k_e$  is the effective thermal conductivity,  $c_p$  specific heat at constant pressure,  $Q$  is the radiation absorption coefficient and  $D$  is the molecular diffusivity,  $k_r$  is the chemical reaction parameter and  $r$  is the radius of the cone.

Following Chamkha [18-19] and Partha [12] and Kameswaran et.al [16] thermophoretic velocity  $v_t$  can be written as

$$v_t = -k \frac{\nu}{T} \frac{\partial T}{\partial z} \quad (7)$$

where  $k$  is the thermophoretic constant,  $T$  is the temperature.

Introducing the following non-dimensional variables to get the non-dimensional governing equations

$$\eta = \left( \frac{\Omega \sin(\alpha)}{\nu} \right)^{1/2} z, \quad u = x \Omega \sin(\alpha) F(\eta), \quad v = x \Omega \sin(\alpha) G(\eta), \quad r = x \sin(\alpha) \quad (8)$$

$$w = (x \Omega \sin(\alpha))^{1/2} H(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

$$T_w(x) - T_\infty = \frac{(T_L - T_\infty)x}{L}, \quad C_w(x) - C_\infty = \frac{(C_L - C_\infty)x}{L}$$

where  $L$  being the cone slant height and  $T_L$  being the cone surface temperature and  $C_L$  being the cone surface concentration at the base ( $x=L$ )

Using (7) and (8), eqns. (1)-(6) reduced into

$$F = -\frac{1}{2} H' \quad (9)$$

$$-H''' + HH' - \frac{H'^2}{2} + Da^{-1}H' + 2G^2 + 2g_s(\theta + N\phi) = 0 \quad (10)$$

$$G'' - HG' + H'G - Da^{-1}G = 0 \quad (11)$$

$$\theta'' + Pr \left( \frac{1}{2} H' \theta - H \theta' + Q_1 \phi \right) = 0 \quad (12)$$

$$\phi'' + Sc \left( \frac{1}{2} H' \phi - H \phi' - \gamma \phi \right) + \frac{Sc k N_T}{N_T \theta + 1} \left( \phi' \theta' + \phi \theta'' - \frac{N_T \phi \theta'^2}{N_T \theta + 1} \right) = 0 \quad (13)$$

The boundary conditions are

$$\begin{aligned} H=0, H'=0, G=1, \theta=1, \phi=1 \quad \text{at } \eta=0 \\ H'=0, G=0, \theta=0, \phi=0 \quad \text{as } \eta \rightarrow \infty \end{aligned} \quad (14)$$

Where  $Da^{-1} = \frac{\nu}{K \Omega \sin \alpha}$  is the inverse of Darcy number,

$Gr = \frac{g \beta_T (T_w - T_\infty) L^3 \cos \alpha}{\nu^2}$  is the Grashof number,  $N = \frac{\beta_c (C_w - C_\infty)}{\beta_T (T_w - T_\infty)}$  is the

buoyancy ratio,  $Re = \frac{\Omega L^2 \cos \alpha}{\nu}$  is the local Reynolds number,

$g_s = \frac{Gr}{Re^2}$  is the mixed convection parameter,  $Pr = \frac{\mu c_p}{k_e}$  is the

Prandtl number,  $Sc = \frac{\nu}{D}$  is the Schmidt number,  $N_T = \frac{T_w - T_\infty}{T_\infty}$  is

the temperature difference parameter  $Q_1 = \frac{Q}{\Omega \sin \alpha} \left( \frac{C_w(x) - C_\infty}{T_w(x) - T_\infty} \right)$  is

the radiation absorption parameter and  $\gamma = \frac{k_r}{\Omega \sin \alpha}$  is the

chemical reaction parameter.

The physical parameters of interest local skin friction

coefficients  $x$  and  $y$ -directions, local Nusselt number, local Sherwood number and wall thermophoretic deposition velocity in non-dimensional form are given by

$$C_{fx} Re^{1/2} = -H''(0) \quad (15)$$

$$2^{-1} Re^{1/2} C_{fy} = -G'(0) \quad (16)$$

$$Re^{-1/2} Nu_x = -\theta'(0) \quad (17)$$

$$Re^{-1/2} Sh_x = -\phi'(0) \quad (18)$$

$$V_t = \left( \frac{-k N_t}{N_t + 1} \right) \theta'(0) \quad (19)$$

### 3 NUMERICAL METHOD

The non-dimensional governing equations (9) - (13) with boundary conditions (14) form highly nonlinear coupled ordinary differential equations which are solved using shooting technique that uses fourth order Runge-Kutta method. In this  $\eta = \eta_{max}$ , condition at infinity, has been suitably chosen at every step such that velocity, temperature and concentration profiles approach the outer edge of the boundary layer. More calculations are performed to find the values of skin-friction coefficient, wall temperature and concentration and wall thermophoretic velocity for a wide range of parameters. The physical parameters, chemical reaction parameter, thermophoretic coefficient and thermophoretic parameters on fluid flow characteristics are studied.

In the absence of concentration equation and inverse Darcy number the reduced eqns. of (9) - (13) similar to Hering and Grosh [5] who investigated combined convection over a rotating cone. It is shown that numerical results of the present method are in good agreement with the results presented by Hering and Grosh [5] as shown in Table 1.

### 4 RESULTS AND DISCUSSION

We have presented graphical solutions for the effects of chemical reaction parameter  $\gamma$  on velocity, temperature and concentration profiles with distance normal to the cone surface. Additionally we computed the variation skin friction coefficient in  $x$  and  $z$  directions, local Nusselt number, local Sherwood number and wall thermophoretic velocity for different values of chemical reaction parameter  $\gamma$ , thermophoretic coefficient ( $k$ ) and temperature difference parameter ( $N_t$ ). The results of this parametric study are shown in Figs. 2-17.

Fig-2 shows the effects of chemical reaction parameter  $\gamma$  on tangential velocity profile ( $F$ ). It is observed from this fig that increasing chemical reaction parameter retards the tangential velocity; this is due to the reduction of fluid behavior near to the cone surface when the chemical reaction parameter increases. Fig-3 & 4 show circumferential and normal velocity profiles for different values of chemical reaction parameter  $\gamma$ . These figures indicate that increasing chemical reaction parameter leads to the enhancement of circumferential and normal velocity profiles for the cause of accelerating fluid motion in circumferential and normal direction by increasing

chemical reaction parameter. Fig 5 & 6 depicts temperature and concentration profiles respectively for different values of chemical reaction parameter respectively. With respect to Fig-6, it is observed that when the chemical reaction parameter increases, temperature profile increases significantly, conversely the concentration profile decreases as increase in the chemical reaction parameter. Therefore we conclude that increase in chemical reaction parameter leads to enhance thermodynamic boundary layer in  $y$  and  $z$  direction and thermal boundary layer while it reduces velocity boundary layer in  $x$ -direction and solution boundary layer thickness.

Table-1: The values of  $-H''(0)$ ,  $-G'(0)$  and  $-\theta'(0)$  for different values of  $g_s$  for  $Pr = 0.7$ ,  $Da^{-1}=0$ , and  $N = 0$  and in the absence of concentration equation.

$g_s$	$-H''(0)$		$-G'(0)$		$-\theta'(0)$	
	Hering and Grosh [5]	Present work	Hering and Grosh [5]	Present work	Hering and Grosh [5]	Present work
0	1.0205	1.0203	0.61592	0.61583	0.42852	0.42842
0.1	1.1369	1.1368	0.65489	0.65492	0.46156	0.46141
1.0	2.2078	2.2075	0.85076	0.85080	0.61202	0.61213
10	8.5246	8.5243	1.40370	1.40363	1.01730	1.01748

Figs 7-10 depict respectively, tangential and azimuthal skin-friction coefficient and local Nusselt number and Sherwood number for different values of chemical reaction parameter. Increasing chemical reaction parameter from  $\gamma = -1$  to  $\gamma = 1$ , tangential and azimuthal skin friction coefficients decreases. Moreover, it is observed that as chemical reaction parameter increases, local Sherwood number increased enormously but the opposite results are reported for local Nusselt number. Figs 11-14 represents respectively, tangential and azimuthal skin friction coefficient, local Nusselt number and Sherwood number for different values of temperature difference parameter  $N_t$  on thermophoretic constant  $k$ . It is evident from these figures that increase in  $N_t$  increases tangential and azimuthal skin friction coefficient and local heat transfer (Nusselt number) rate but reduces mass transfer rate (local Sherwood number) significantly. Moreover, it is observed that increase in thermophoretic constant  $k$ , there is an enhancement in tangential and azimuthal skin-friction coefficient and local Nusselt number while it reduces local Sherwood number.

Figs 15-17 depict respectively, thermophoretic velocity  $V_{tw}$  near the cone surface for different values of chemical reaction parameter  $\gamma$ , thermophoretic constant and temperature difference parameter  $N_t$ . As chemical

reaction parameter  $\gamma$  increases, wall thermophoretic velocity increases (**Fig-15**). From **fig-16**, it is noticed that when temperature difference parameter  $Nt$  increases, there is significant depreciation in wall thermophoretic velocity. From **fig-17**, we perceive that increase in thermophoretic constant  $k$  decreases thermophoretic wall deposition velocity  $V_{tw}$  significantly. Moreover it is clearly seen that wall thermophoretic velocity decreases when increase in mixed convection parameter. These variations in thermophoretic wall deposition velocity will be significant in industrial and engineering application.

## 5 CONCLUSIONS

Numerical solutions are reported to analyze the chemical reaction and thermophoretic effect on rotating vertical cone in a fluid saturated Darcy porous medium. The effect of chemical reaction parameter on velocity profile in  $x$ ,  $y$  and  $z$  direction and temperature and concentration profile is analyzed as well as skin friction coefficient  $x$  and  $z$  directions, local Nusselt number, local Sherwood number and wall thermophoretic deposition velocity is analyzed for different values of  $\gamma$ ,  $Nt$  and  $k$ . It is worth mentioning that destructive chemical reaction is more influence than generative chemical reaction parameter. As chemical reaction parameter increases, tangential velocity, concentration profiles and tangential and azimuthal skin friction coefficients and local Nusselt number decreases but circumferential and normal velocity, temperature profiles and local Sherwood number values are increased significantly. Increase in temperature difference parameter enhances tangential and circumferential skin-friction coefficients and local Nusselt number while it reduces local Sherwood number, as well as thermophoretic constant reported opposite results with those of temperature difference parameter. Wall thermophoretic deposition velocity increases when chemical reaction parameter increases but it produces opposite results for larger values of thermophoretic constant, temperature difference parameter and mixed convection parameter.

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**GRAPHS**





